Spectra of turbulence in a round jet

By M. M. GIBSON

Department of Mechanics, The Johns Hopkins University, Baltimore, Maryland†

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Turbulent energy spectra have been measured in a round air jet at high Reynolds numbers ($R_\lambda = 710$ to 780). The streamwise and cross-stream spectrum functions are well fitted by the $(-\frac{5}{3})$ power law predicted by the Kolmogoroff theory for an inertial subrange. The data are shown to be consistent with the local isotropy hypothesis and the constant in the Kolmogoroff spectrum function in the inertial subrange is evaluated. Effects of length and non-linear response of the hot-wire are considered.

1. Introduction

The Universal Equilibrium Theory, due to Kolmogoroff (1941), has long been accepted as an important part of contemporary turbulence theory. Direct confirmation by experiment has, however, been hampered by the difficulty of obtaining high enough Reynolds numbers in laboratory flows for the theory to be applicable. Although several authors, for example, Townsend (1948 *a*, *b*), Corrsin (1949) and Laufer (1954) have obtained data consistent with statistical isotropy of the small-scale turbulence, comparatively little data are available to confirm the existence of a universal range in the turbulent energy spectrum.

Grant, Stewart & Moilliet (1962), in presenting their recent spectrum measurements in a tidal channel, have reviewed the Kolmogoroff theory, some of the semi-empirical theories for the form of the energy spectrum and the limited amount of experimental data. In notation similar to that of these authors a three-dimensional spectrum function $E(k)$ is defined (see Hinze 1959 or Batchelor 1953),

$$\int_0^\infty E(k)\,dk = \tfrac{1}{2}\overline{q^2} = \tfrac{1}{2}(\overline{u^2}+\overline{v^2}+\overline{w^2}), \tag{1.1}$$

where u, v and w are turbulent velocity components which, in the round jet, can be taken in the longitudinal, radial and circumferential directions respectively. Harmonic analysis of a hot-wire signal yields only the one-dimensional spectrum functions: $F_1(k)$, $F_2(k)$, $F_3(k)$, where

$$\int_0^\infty F_1(k)\,dk = \overline{u^2}, \tag{1.2}$$

$$\int_0^\infty F_2(k)\,dk = \overline{v^2}, \tag{1.3}$$

$$\int_0^\infty F_3(k)\,dk = \overline{w^2}, \tag{1.4}$$

† Present address: Northern Research and Engineering Corporation, 195 Knightsbridge, London, S.W. 7.

which, for isotropic turbulence, are related to $E(k)$ by

$$E(k) = -k\frac{d}{dk}\{\tfrac{1}{2}F_1(k) + F_2(k)\} \tag{1.5}$$

(Hinze 1959), and to each other by

$$\frac{d}{dk}F_2(k) = \frac{d}{dk}F_3(k) = -\frac{k}{2}\frac{d^2}{dk^2}F_1(k), \tag{1.6}$$

where k is a wave-number component corresponding to the streamwise direction, $\overline{u^2} = \overline{v^2} = \overline{w^2}$ and $F_2(k) = F_3(k)$ in isotropic turbulence.

The rate of turbulent energy dissipation in isotropic turbulence is given by

$$\epsilon = 2\nu\int_0^\infty k^2E(k)\,dk \tag{1.7}$$

$$= 15\nu\int_0^\infty k^2F_1(k)\,dk, \tag{1.8}$$

where ν is the kinematic viscosity.

In these terms the Universal Equilibrium Theory asserts that, at wave-numbers high enough for the small-scale turbulence to be statistically independent of the large-scale motion containing most of the turbulent energy, the turbulence is locally isotropic and the energy spectrum function has a universal form determined only by the two parameters ν and ϵ given by

$$E(k) = u_k^2\eta E_k(k\eta) \tag{1.9}$$

(Batchelor 1953), where u_k and η are the Kolmogoroff velocity and length microscales

$$u_k = (\nu\epsilon)^{\frac{1}{4}}; \quad \eta = (\nu^3/\epsilon)^{\frac{1}{4}}, \tag{1.10}$$

and $E_k(k\eta)$ is a universal function.

If the energy containing and the dissipation wave-number ranges are widely separated, so that energy dissipation takes place almost wholly within the equilibrium range, then the theory postulates an inertial subrange at the lower end of the universal range where energy transfer through the spectrum is independent of viscosity and $E(k)$ takes the form

$$E(k) = A\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}}. \tag{1.11}$$

Hence from (1.5), (1.6), with local isotropy,

$$F_1(k) = A_1\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}}, \tag{1.12}$$

$$F_2(k) = F_3(k) = A_2\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}}, \tag{1.13}$$

where

$$A_1 = \tfrac{18}{55}A, \tag{1.14}$$

$$A_2 = \tfrac{4}{3}A_1 = \tfrac{24}{55}A. \tag{1.15}$$

A convenient criterion for the probable existence of an inertial subrange is the magnitude of the Reynolds number R_λ based on the root-mean-square turbulent velocity and the Taylor lateral microscale λ. Corrsin (1958) has indicated that values of R_λ greater than 500 are probably required to establish an inertial subrange.

It was the purpose of this investigation to provide a detailed experimental check on local isotropy and on the $-\frac{5}{3}$ power-law dependency of the energy spectra in an inertial subrange. Hot-wire anemometer measurements were made in the highly turbulent and non-isotropic flow in a round air jet for which R_λ was *a priori* estimated to be 550.

2. Experimental equipment

The round jet

The jet was generated by two $1\frac{1}{2}$ h.p., 3500 r.p.m. single-stage axial fan units in tandem exhausting through a nozzle with diameter $15\frac{1}{4}$ cm and contraction ratio 20 : 3. The initial total pressure was maintained at approximately 6·4 in. water gauge, the maximum deviation from this value not exceeding 5 % for each run.

Measuring equipment

Mean and turbulent velocities were measured with a modified version of the Hubbard II HR Type 3 A Linear Constant-Temperature Hot-Wire anemometer (Hubbard 1957). The constant-temperature mode of operation and the linear response of this instrument renders it particularly suitable for measurements at high turbulence levels.

Amplifiers in the bridge circuit are direct coupled with feedback to allow direct mean-velocity measurements and, although sensitive to temperature change, are quite stable in operation. The Hubbard r.m.s. Analyser, a servicing unit containing additional amplifier stages and adding, subtracting and squaring circuits, is a.c. coupled with a flat frequency characteristic enabling it to be used down to 1 c/s.

The upper frequency limit was determined by the thermal noise level which was greatly dependent upon the amount of feedback in the first amplifier stage and thus could not be measured directly and subtracted from the total signal. Approximate noise spectra were measured by analysis of the signals from a hot-wire in a very low-turbulence airstream at mean velocities equal to those found in the jet. The upper cut-off was taken to be the frequency where this noise level ceased to make a negligible contribution to the total signal, that is at about 7 kc/s. This method was not sufficiently accurate to allow the spectra to be corrected for noise at frequencies much higher than this.

Tungsten wires 0·00015 in. in diameter, copper plated and etched to 5 Ω cold resistance, with length approximately 1·0 mm, formed the sensing element. To reduce temperature sensitivity they were operated at the relatively large overheat ratios of 0·5 to 0·8. Mean velocities and streamwise turbulent velocities were measured with a single wire probe. An X-probe was used for measurements of the cross-stream components of the turbulence and the turbulent shear stress.

In view of the high turbulence intensities and low frequencies encountered in the jet, long time averaging of the direct current and squared alternating-current signals was necessary; this was achieved using Philbrick operational amplifiers in the manner of analogue computing.

Wave analysers and filters

Harmonic analysis of the alternating current signal from the hot-wire was performed, in the range 1 to 100 c/s, by a Krohn–Hite 330 A continuously variable band-pass filter and, for frequencies greater than 20 c/s, by a slightly modified Hewlett–Packard 302 A Wave Analyser.

The equivalent band width of the Krohn–Hite Filter is 0·54 times the mid-band frequency; that of the Hewlett–Packard Analyser has a fixed value of 6 c/s over the entire range. Measurements with the two instruments in the overlap range 20–100 c/s were in excellent agreement and it can be concluded that the effect of averaging the highly non-uniform spectrum function over the rather wide pass band of the filter was negligible.

Vacuum thermocouple squaring circuits and Philbrick analogue computer elements were used for time averaging of the filter and analyser output signals.

3. Experimental results

Survey measurements

Profiles of mean velocity, turbulent velocity components and turbulent shear correlations were measured on a jet diameter fifty orifice diameters from the nozzle ($x/d = 50$) and are plotted in figures 1 and 2. The mean-velocity profile is slightly asymmetric and rather fuller than might be expected for self-preserving free-jet flow (Townsend 1956). This may be attributed to some unavoidable interference from the walls and floor of this laboratory.

Energy and dissipation spectra

Figures 3–5 are logarithmic plots of the one-dimensional energy spectrum functions $F_1(k)$, $F_2(k)$ and $F_3(k)$ measured both on and at 40·5 cm ($r/r_{\frac{1}{2}} = 0{\cdot}5$, where $r_{\frac{1}{2}}$ = 'half-velocity radius') from the jet axis in the plane $x/d = 50$.

It was assumed (Taylor 1938) that wave-number and frequency are related by

$$k = \frac{2\pi n}{\overline{U}}, \tag{3.1}$$

where $2\pi n$ is the angular frequency in radians per second and $\overline{U}$ is the mean velocity.

Semi-logarithmic plots of the dissipation spectra $k^2F_1(k)$ (figure 6) shows that the energy-containing and dissipation wave-number ranges are fairly well separated, and that the fundamental condition for an inertial subrange may be at least approximately satisfied.

The spectra are all reasonably well fitted by a $-\frac{5}{3}$ power law in k as predicted by the Kolmogoroff theory. The extent of the $-\frac{5}{3}$ range is about two decades of k for the longitudinal spectra and rather less for the lateral.

On the jet axis the root-mean-square turbulent velocity components were very nearly equal. Also $F_2(k) = F_3(k)$ through symmetry and the shear correlations $\overline{uv} = \overline{uw} = 0$. Thus the motion had here some features characteristic of isotropic turbulence. At the off-axis position the turbulence was highly non-isotropic and the shear correlation reached its maximum value. These measurements therefore are well suited for checking the concept of local isotropy at the higher wave-numbers.

Measurements of the shear correlation coefficient

$$R_{u_1v_1}(k) = \frac{\overline{u_1v_1}}{u_1'v_1'} \tag{3.2}$$

are plotted in figure 6, where $\overline{u_1v_1}$ is the contribution to the total shear correlation $\overline{uv}$ and u_1', v_1' the half powers of the contributions to the one-dimensional spectrum functions $F_1(k)$, $F_2(k)$ associated with wave-number k (Corrsin 1949; Corrsin &

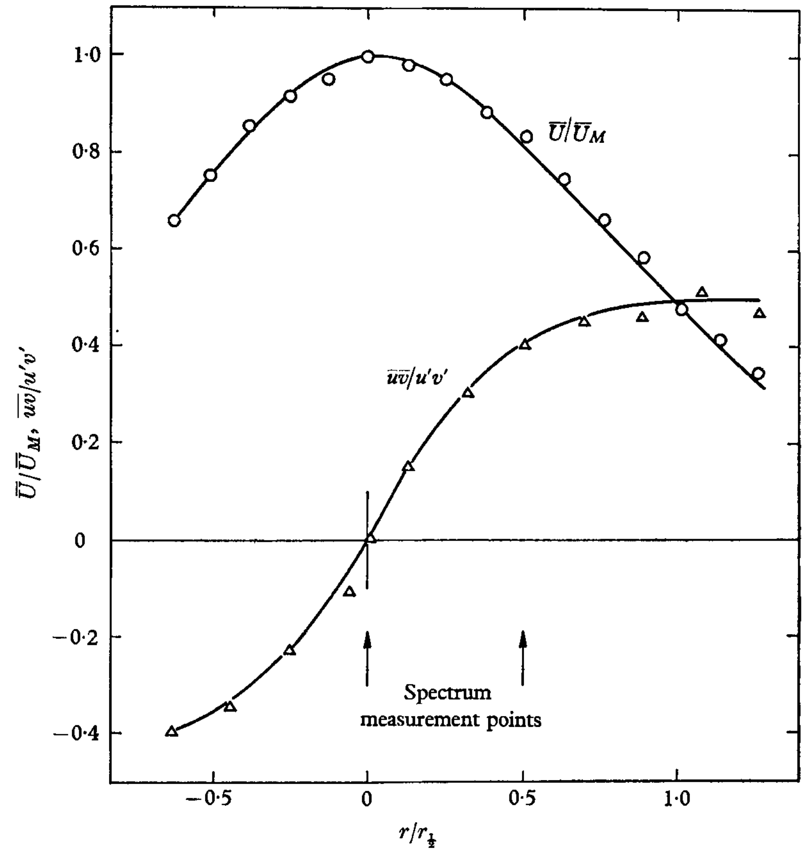

FIGURE 1. Radial distribution of mean velocity and shear correlation coefficient. $x/d = 50$. Mean velocities corrected for non-linear hot-wire response are faired by solid line.

Uberoi 1951). The function decreases monotonically to zero at comparatively low wave-numbers with the inference that the motion is locally isotropic for, approximately, $k > 0{\cdot}4\,\mathrm{cm}^{-1}$.

A less direct check is to use the isotropic relations (1.6) to derive one one-dimensional spectrum from another. To avoid errors inherent in the differentiation of experimentally determined functions the integral form of (1.6) was used to derive $F_1(k)$ from the measured $F_2(k)$ and $F_3(k)$ with the results plotted, for the off-axis position, in figure 7. In the universal range the lateral spectra should, of course, coincide. The spread of the data in the $k^{-\frac{5}{3}}$ range may reasonably be attributed to calibration errors not affecting the shape of the spectrum functions.

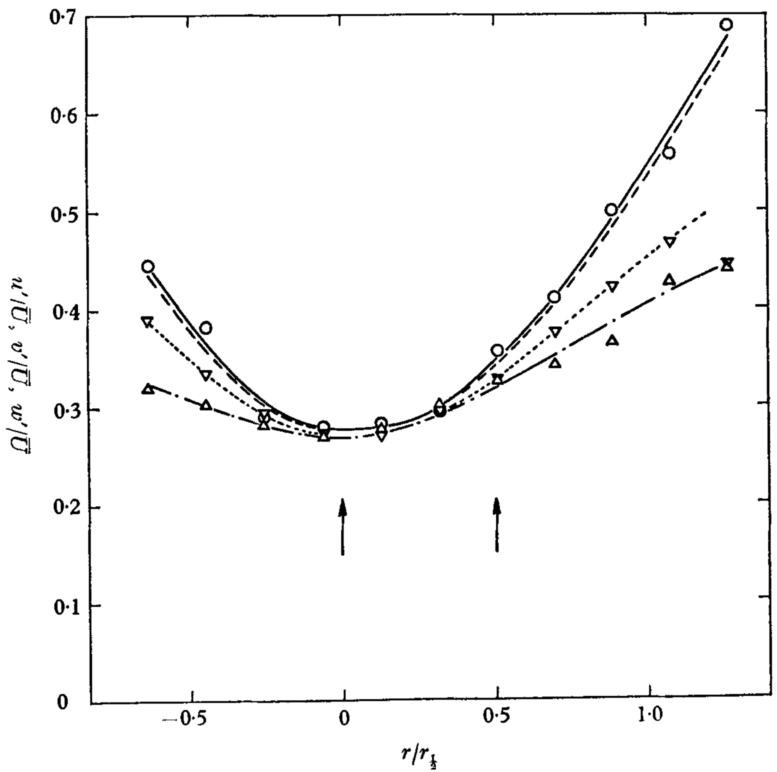

FIGURE 2. Radial distributions of r.m s. turbulence intensities. ○, $u'/\overline{U}$; △, $v'/\overline{U}$; ▽, $w'/\overline{U}$; ----, $u'/\overline{U}$ corrected for non-linear hot-wire response.

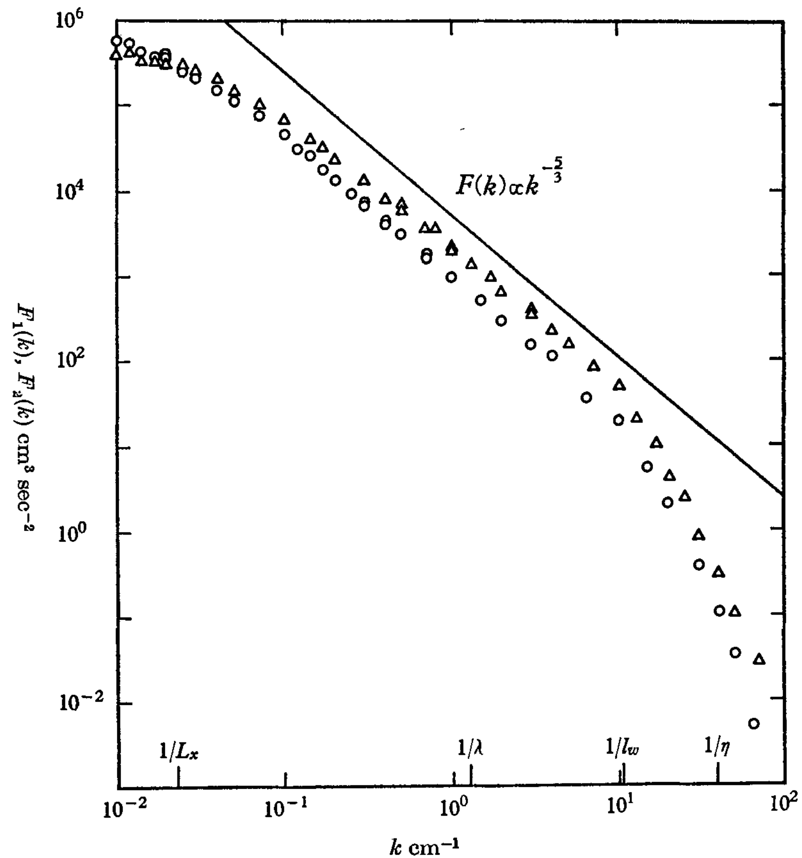

FIGURE 3. $F_1(k)$ and $F_2(k)$ measured on the jet axis. l_w = hot-wire length. ○, $F_1(k)$; △, $F_2(k)$.

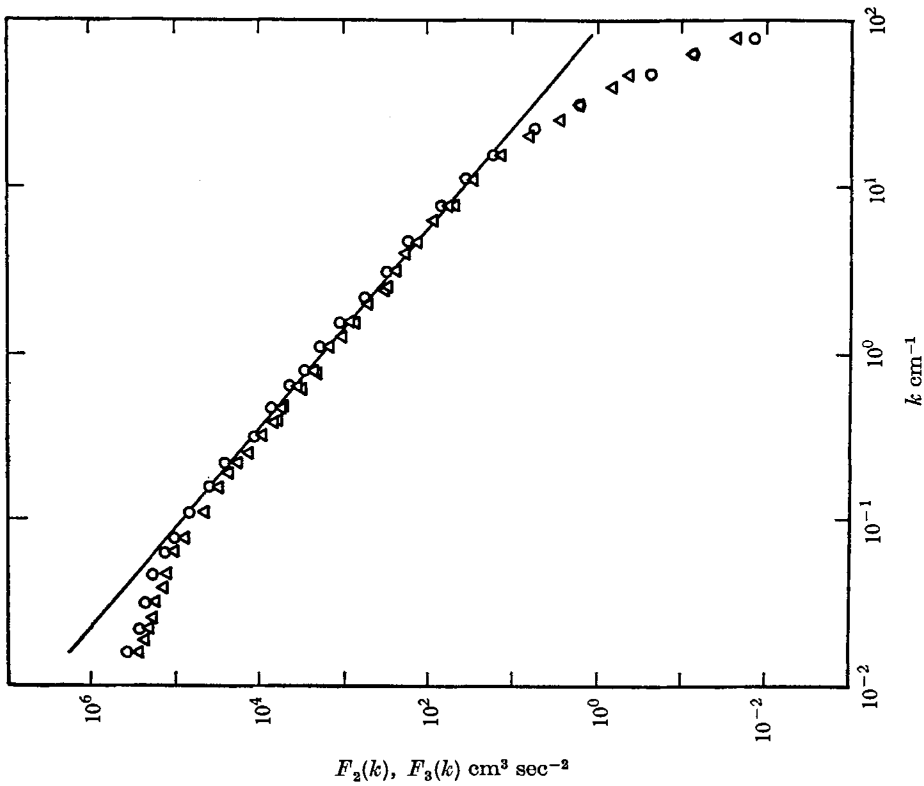

FIGURE 5. $F_2(k)$ and $F_3(k)$ measured at $r/r_{\frac{1}{2}} = 0{\cdot}5$. ○, $F_2(k)$; △, $F_3(k)$.

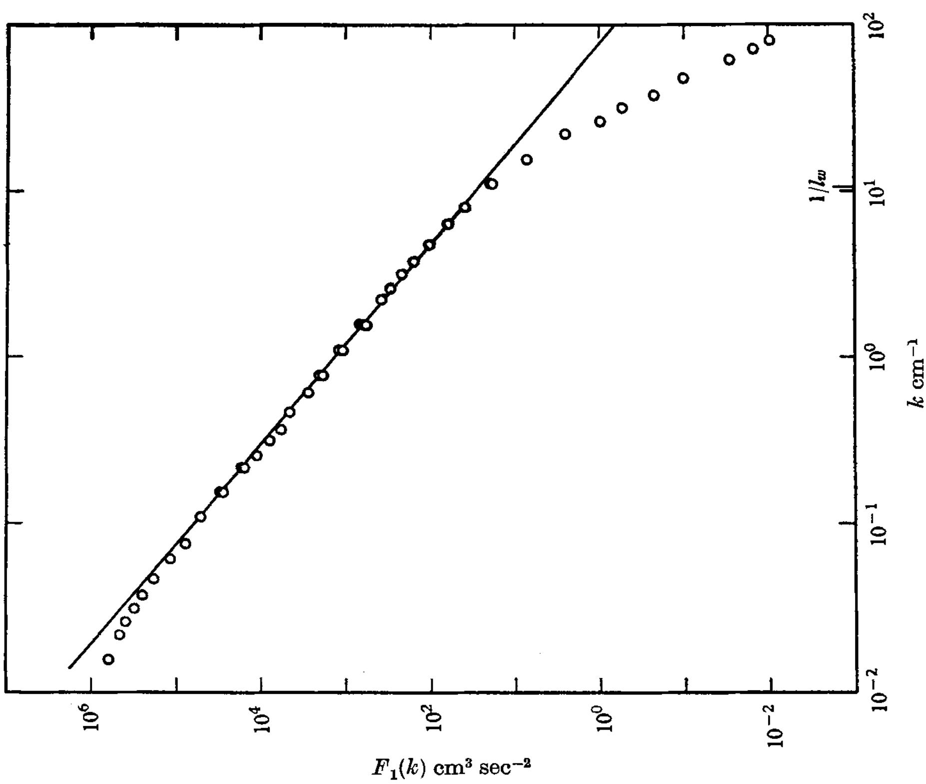

FIGURE 4. $F_1(k)$ measured on the axis at $r/r_{\frac{1}{2}} = 0{\cdot}5$.

The agreement between the derived and measured values of $F_1(k)$ is considered reasonable and, although not a conclusive test, consistent with the local-isotropy concept. The same method applied to the axis measurements yielded very similar results (Gibson 1962).

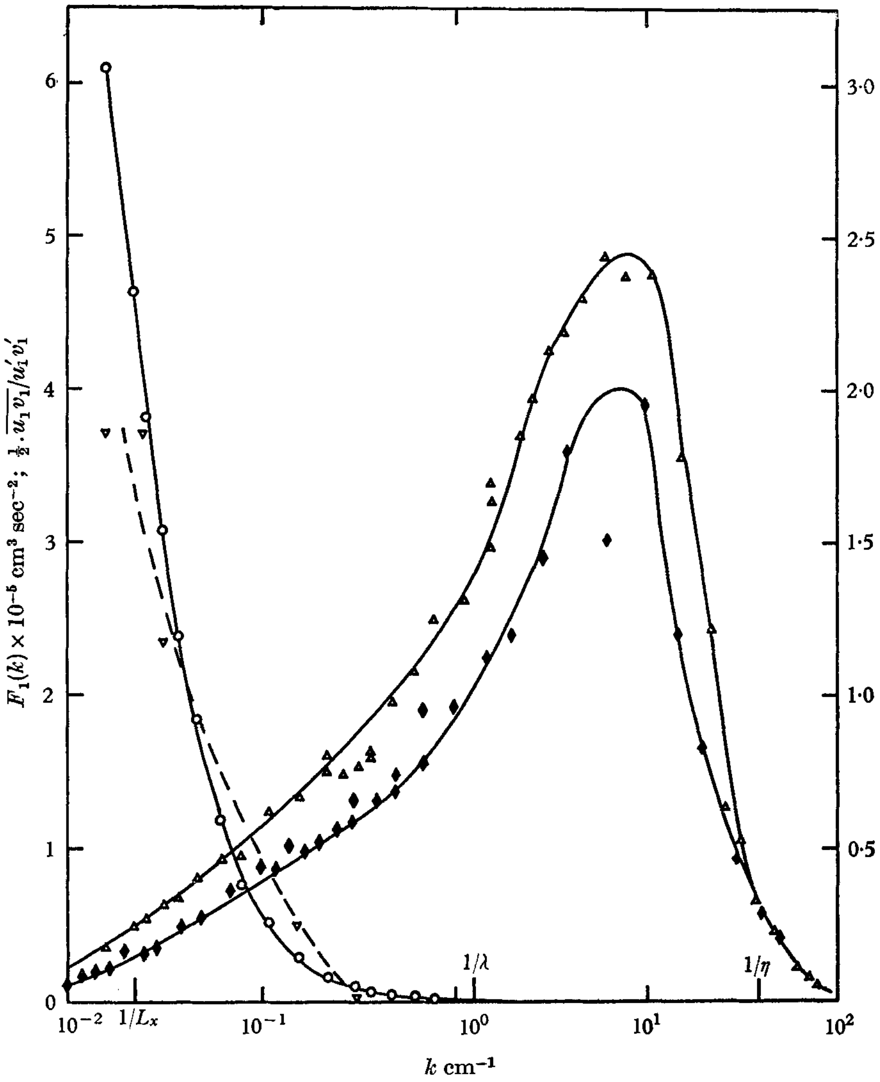

FIGURE 6. Energy, dissipation and Reynolds-stress correlation-coefficient spectra. On axis: ◆, $k^2F_1(k)$. Off-axis: ○, $F_1(k)$; △, $k^2F_1(k)$; ▽, $\overline{u_1v_1}/u_1'v_1'$.

On the assumption, based on the above results, that the motion associated with most of the turbulent energy dissipation was locally isotropic, equation (1.8) was used to compute the dissipation rates. Isotropic turbulence relations, although not strictly valid in this case, were used to calculate other characteristic

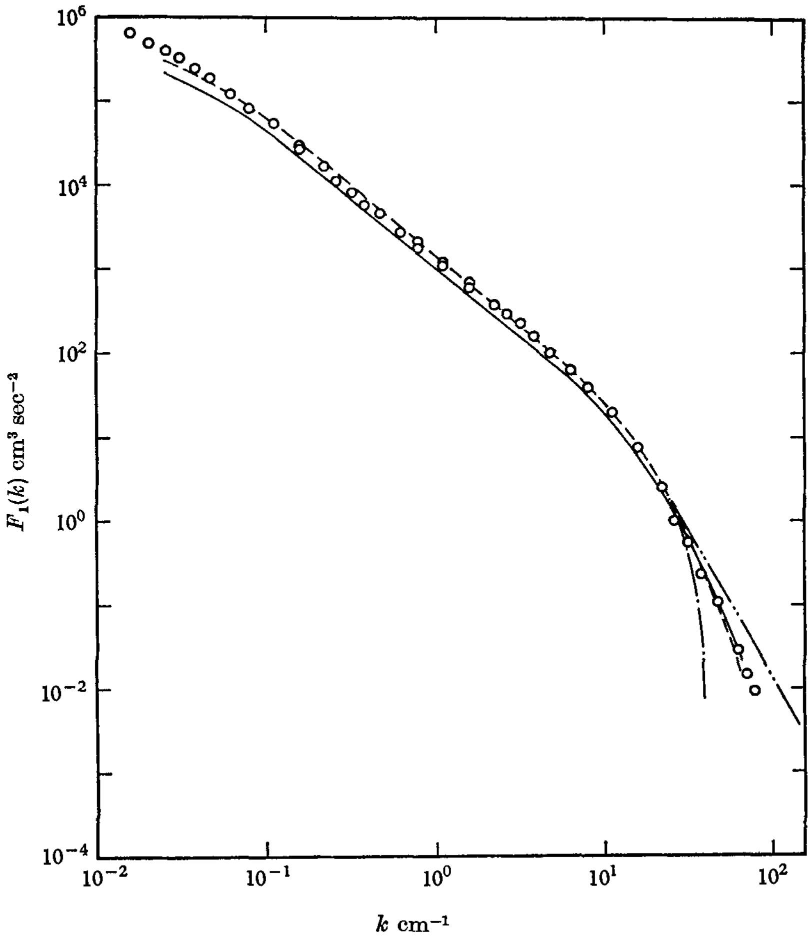

FIGURE 7. Comparison of measured $F_1(k)$ with those derived from $F_2(k)$ and $F_3(k)$ using the isotropic relations. Off-axis position, ○, $F_1(k)$ measured; ——, $F_1(k)$ from $F_2(k)$; ----, $F_1(k)$ from $F_3(k)$; —·—, Kovasznay spectrum function; —··—, $F_1(k)$ corrected for wire length.

Position	Axis	Off-axis
$(r/r_{\frac{1}{2}})$	0	0·5
ϵ cm^2 sec^{-3}	86×10^3	133×10^3
η cm	$1{\cdot}41 \times 10^{-3}$	$1{\cdot}26 \times 10^{-3}$
λ cm	0·80	0·66
L_x cm	61·5	61·5
R_λ	780	710
A	1·57	1·62

TABLE 1

parameters, the Kolmogoroff microscale from (1.10) and the Taylor microscale and the integral scale from

$$\left.\begin{aligned} \frac{\frac{1}{3}\overline{q^2}}{\lambda^2} &= \int_0^\infty k^2 F_1(k)\,dk, \\ L_x &= \frac{\pi}{2}\frac{F_1(0)}{\overline{u^2}}, \end{aligned}\right\} \tag{3.3}$$

as given by Hinze (1959). The Kolmogoroff constant A, for the inertial subrange, was obtained from the single-wire measurements of $F_1(k)$ considered to be rather more accurate than the X-wire lateral spectra measurements. Values of these quantities are presented in table 1.

4. Sources of error and corrections

Wire-length corrections

The limited spatial resolution of the hot-wire with respect to turbulence whose length scales are comparable to the wire dimensions makes it necessary to apply some correction to the measured spectra. The effect of wire length on the measured longitudinal spectrum $F_1(k)$ has been analysed by Uberoi & Kovasznay (1953) with the assumption of uniform wire heating. To apply this type of correction to the present results the data were fitted by a Kovasznay (1948) spectrum function and a power law (figure 7), and, using these functions, the wire length correction was found to be quite negligible for $k < 15\,\text{cm}^{-1}$ and about 25 % at $k = 25\,\text{cm}^{-1}$. The wire length was taken to be 0·1 cm. The corrected spectrum is plotted in figure 7.

The $k^{-\frac{5}{3}}$ range is affected only by a change in the constant A. The corrected spectrum results in a 15 % increase in ϵ and a corresponding decrease in A of 10 %. The same method was applied to the axis measurements with almost identical results.

The analysis cannot readily be applied to correction of cross-stream spectra, but there is little doubt that the lengths of the x-probe being roughly equal to that of the single wire the corrections should be applied in the same wave-number range and have a similar form.

Non-linear wire response

While systematic errors associated with the non-linear response of the hot-wire to the amplitude of the turbulence may be eliminated by appropriate electronic circuitry in conjunction with a constant-temperature hot-wire, the instrument retains a non-linear response to directional fluctuations and if, as in the jet, these are large the resulting errors may be significant. This effect has been examined by Rose (1962) who derived corrections for mean and turbulence velocities which have been applied to the present measurements with the results shown in figures 1 and 2.

To make a rough estimate of this effect on the shape of the spectrum the simple functions

$$u/\overline{U} = a_1 \cos \omega t + a_2 \cos (2\omega t - \theta), \tag{4.1}$$

$$v/\overline{U} = b_1 \cos (\omega t - \phi) + b_2 \cos (2\omega t - \psi), \tag{4.2}$$

are inserted in Rose's expression for the instantaneous a.c. hot-wire response to the second order in $u/\overline{U}$, $v/\overline{U}$, namely

$$\frac{i}{\overline{I}} = \frac{u}{\overline{U}_c} + \frac{1}{2}\left[\frac{v^2}{\overline{U}_c^2} - \frac{\overline{v^2}}{\overline{U}_c^2}\right], \tag{4.3}$$

where i and $\overline{I}$ are the instantaneous fluctuating and mean wire currents respectively and the subscript c denotes calibration conditions in a flow of zero turbulence. The mean square of the response to the second harmonic then turns out to be

$$\left(\frac{\overline{i^2}}{\overline{I}^2}\right)_{2\omega t} = \frac{1}{2}\left[a_2^2 + \frac{a_2 b_1^2}{2}\cos(2\theta - \phi) + \frac{b_1^4}{16}\right]. \tag{4.4}$$

The measured spectrum at this point is thus in error, for this simplified case, by a factor

$$\delta_{2\omega t} = 1 + \frac{b_1^2}{a_2}\cos(2\theta - \phi) + \frac{b_1^4}{16 a_2^2}. \tag{4.5}$$

An order of magnitude estimate of this quantity for the actual experimental data may be made by equating the coefficients a, b to the energy per unit frequency in the spectrum. This is equivalent to considering the spectrum to be composed of a number of discrete Dirac functions each corresponding in area to a finite bandwidth. Then, taking $F_1(k) = 10^4\,\text{cm}^3\,\text{sec}^{-2}$ as the maximum value in the $k^{-\frac{5}{3}}$ range and assuming local isotropy

$$\delta_{2k} = 1 + 0{\cdot}05\cos(2\theta - \phi) + 1{\cdot}75 \times 10^{-4}. \tag{4.6}$$

At lower wave-numbers outside the inertial subrange, if actual values of $F_1(k)$ and $F_2(k)$ are substituted for $k = 0{\cdot}016\,\text{cm}^{-1}$, $2k = 0{\cdot}032\,\text{cm}^{-1}$,

$$\delta_{2k} = 1 + 0{\cdot}1\cos(2\theta - \phi) + 6{\cdot}25 \times 10^{-4}. \tag{4.7}$$

The term $\cos(2\theta - \phi)$ is associated with the triple correlation and it seems probable that it averages to zero in locally isotropic turbulence. Thus the corrections to the spectrum for large directional fluctuations are almost certainly negligibly small in the wave-number range of greatest interest but may amount to the order of 10% in the second harmonic for the energy-containing range from this greatly simplified model.

The cross-stream spectra $F_2(k)$, $F_3(k)$ are unaffected both in total level and shape by this trigonometric non-linearity.

Other sources of error

Further errors arise due to use of the constant temperature hot-wire in an environment not temperature controlled. The corrections for mean temperature changes are straightforward (Rose 1962) and have been applied to the data. Previous preliminary measurements with a constant-current hot-wire anemometer (Gibson 1962) indicated that mean-square temperature fluctuations in the jet were negligible compared to the turbulence signal.

It should be pointed out that the validity of the Taylor hypothesis (3.1) is rather doubtful on account of the high turbulence levels encountered in the jet particularly in the shear region (Lin 1953; Uberoi & Corrsin 1953).

5. Discussion

The results tend to support the Universal Equilibrium hypothesis particularly when considered in the light of measurements reported by other authors.

Much of the published data has, of necessity, been obtained from flows for which the Reynolds numbers were too low for the existence of an inertial sub-range. For atmospheric turbulence the work of Obukhov (1941) supported the theory indirectly, and more recently Gurvich (1960) demonstrated the existence of a $-\frac{5}{3}$ power law range in a number of vertical wind frequency spectra. The dependence of the energy dissipation on Richardson number in the stratified atmosphere does not, unfortunately, allow of a direct comparison with the present results.

The tidal channel measurements reported by Grant *et al.* (1962) and Grant & Moilliet (1962) were carried out at very high Reynolds numbers and showed an extensive range for which the spectra obeyed a $-\frac{5}{3}$ power law. Absolute values of the streamwise spectrum function were obtained but not for the cross-stream spectrum.

Flow	R_λ	A
Jet: axis	780	1·57
off-axis	710	1·62
Tidal channel	3600	1·44
Grid turbulence	750	2·70

TABLE 2

Data obtained by Kistler & Vrebalovitch (1961) for grid turbulence in the California Institute of Technology's late Co-operative wind-tunnel showed a $-\frac{5}{3}$ power dependency in the streamwise spectrum which was however not evident in the cross-stream spectrum where a power law $k^{-\frac{3}{2}}$ gave a better fit. These results are similar, in this respect, to those obtained by Laufer (1954) whose Reynolds number, for pipe flow, was comparatively low. Most unfortunately the Co-operative wind-tunnel was dismantled immediately following the experiments and the authors had no opportunity of checking these unexpected results.

The values of the Kolmogoroff constant A obtained from the streamwise one-dimensional spectra measured by Grant *et al.* and Kistler & Vrebalovitch are tabulated in table 2.

The consistency of the values of A from such widely different sources provides, apart from the singular behaviour of the cross-stream grid turbulence spectrum, very strong confirmation of the theory.

The marked intermittency of the high-frequency turbulent fluctuations, observed in the present case and by many other authors, has raised some doubts as to the validity of an average dissipation rate in this context. Kolmogoroff (1962) has refined his original hypothesis to some extent with this in mind, but Grant *et al.* (1962) have indicated, by a simple model, that intermittency in the dissipation is negligible to the accuracy of the reported measurements.

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